## How Can We Teach Thinking Strategies For Basic Addition Facts?

## How can you break the pattern of children counting all to add?

It is natural for children to count all as they begin adding. By helping them learn more efficient ways to think, you can help them make sense of adding with the harder basic facts and mental computation with larger numbers. Helping students move beyond counting is one of the first steps in building their competence and confidence in mathematics.

Why do children continue counting from one to solve basic addition fact problems? From the time children begin to learn about numbers, they have been encouraged to count to find out how many. Furthermore, they have had success with counting to find these answers. As long as they can see the objects or directly model the problem on their fingers, they will tend to use the same counting patterns. This is one of the reasons that students using a manipulative activity program sometimes continue to count all for an extended time. If the counters are right there in front of them, why not count them? The children are confident that they can correctly answer the question using that procedure.

In order to break this pattern of behavior, teachers need to structure experiences where it is much easier to use a different thinking strategy. For example, if you create a situation where the students can not count all, they will try to find different ways to solve the problem.

Two teaching strategies tend to encourage children to look for ways other than counting all to solve the problems. Hiding the large part that is to be added, then showing the small part encourages children to count on. Similarly, using situations where one of the numbers is much larger makes it very difficult to count all. For example, if you have $34 \phi$ in your piggy bank and put another $2 \phi$ in the bank, how much is there now. Counting from 1 to 34 is not efficient, so students try counting on.

## How can you teach counting on for addition?

- Create a word problem where one of the parts being added is small (1, 2, or 3 ).
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they counted on. If not, then tell the students that you heard a student solve it by counting on, then explain how they were thinking. For $6+2$, think 6, ... then count on two more, ... 7, 8.
- You may want to model the counting on thinking by showing the large part, then covering it and reminding the students how many there are, then add the small part one at a time as you count on to get the whole.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use counting on to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use counting on. Besides the routine described in the bullets above, sometimes do the following.

- Ask what addition problem they did and how it could be written as an equation.
- Ask students to start at 3 and count on 8 and compare it to starting at 8 and counting on 3. Reinforce the efficiency of starting with the larger number and counting on the smaller number.
- Discuss when counting on can be used efficiently, that is, when one of the parts being added is small.
- After the students can count on, a few minutes of practice on a regular basis for two or three weeks will enable them to solve counting on problems in about 3 seconds. If they also know the zero generalization, they will be able to solve 64 of the 100 basic addition facts quickly.

Alternative procedures can also be used. One teaching strategy is to use counters to show addition problems, but keep the large set hidden. Then add the small part, one counter at a time. The following are examples of situations that can be created.

Show 6 bear counters. Cover them with your hand as you tell the students that 6 bears went into a cave. Now tell them that 2 more bears went into the cave as you show two more bear counters being moved under your hand. Move them under your hand one at a time. Then ask how many bears are in the cave? Have the students explain how you can count on two more by saying 7,8 . Then uncover the bears to check your answer. Ask what addition problem you just solved. You may want to write an equation to show it, then repeat how counting on can help solve that equation. Repeat with other numbers.

Put 8 counters into a plastic cup. Tell the students how many are there. Now ask them to listen as you drop some more counters into the cup. Drop 1, 2, or 3 more into the cup one at a time. Then ask how many counters are in the cup? Have the students explain how you can count on to solve the problem. Ask the students what addition problem you just solved. You may want to write an equation to show it, then repeat how counting on can help you solve that equation. Repeat with other numbers. To create more mystique, turn your back to the students and tell them you are not going to tell how many more you added. Drop 1, 2, or 3 more counters into the cup one at a time. Act surprised when they can still solve the problem.

Similar situations can be created by saying a number or rolling a number cube, then rolling a dot cube (1, 2 , or 3 dots on each side) to create situations where counting on can easily be used. Ask how many there are in all. Have the students verbalize counting on to show how it can be used to solve the problem. Ask them what addition problem they just solved and write that equation. Repeat with other numbers.

Create other situations where the large part is hidden and children can easily count on the small part being added to it. These can be done with dot cards, rolling dot cubes, putting pennies into a piggy bank or your pocket, having some children join another group that can not be seen, etc.

## How can you teach using doubles for addition?

- Create a word problem where the parts being added are close together, that is, have a difference of one or two.
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they used a known double fact to help them solve the problem.

If not, tell the students that you heard a student solve it by using a double, then explain how they were thinking. For $7+6$, think $6+6$ is 12 , so $7+6$ is 1 more or 13 .

- You may want to model the double thinking by showing a the double then adding one or two (removing one or two).
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use a known double fact to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use doubles. Besides the routine described in the bullets above, sometimes do the following.

- Ask what addition problem they did and how it could be written as an equation.
- Ask students what double could be used to help you solve problems, such as, $5+6(5+$ 5 or $6+6) ; 8+6(6+6,8+8$, or $7+7)$
- Discuss when doubles can be used efficiently, that is, when the parts are close together. Compare these problems to counting on problems where one of the numbers is small. Have the students choose whether they would count on or use doubles to solve several different problems.
- After the students can use doubles, a few minutes of practice on a regular basis for two or three weeks will enable them to solve doubles and near doubles problems in about 3 seconds. This will enable them to solve 24 additional problems beyond the 64 they can do because of the zero generalization and counting on. They can now easily solve 88 of the 100 addition facts.

Before students can use doubles to help them solve near-double facts, they obviously need to know some of the doubles. However, they do not need to know all of the doubles before they start learning this strategy. For example, most children already know that $5+5$ is 10 . That can be a good starting point for this strategy.

In order to help students learn to use doubles, start with a double, show both parts, ask how many there are in all, then add one more counter to one of the parts, repeat the double fact and ask the new fact that is shown. For example, show 5 and 5 . Ask what the total is. Then put one more counter with one of the parts. Say five and five make ten, how much is 6 and 5 ? The students can easily see that only one more counter was added to the total, so they know the answer is just one more than ten.

Create examples like that described in the paragraph above. Each time show a double, add (or remove) one or two more counters, repeat the double fact, and ask how many there are now. Just like with counting on, students are more likely to use the information that is given and compensate for the change if the numbers are large. If the numbers are small, the students may tend to count them all.

Write several near doubles problems on the board. Ask which doubles can be used with each of the problems. Note that there is not a single answer for any of these problems. The smaller number can be doubled, then you can count on one or two more. Or, the larger number can be doubled, then you can count back one or two. Or, if there is a difference of two, some students learn to double the middle number. One group of second graders called this the Robin Hood method. In each case, the thinking can be illustrated with counters to show others in the class exactly what a student is thinking.

## How can you teach making ten for addition?

- Create a word problem where one of the parts being added is an 8 or a 9 and the other makes the total more than 10.
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they added to make ten, then added the rest to get the total. If not, tell the students that you heard a student solve it by making ten. For $9+6$, think 9 and 1 is 10 , then 5 more is 15 .
- You may want to model the make ten thinking by showing the 8 or 9 objects in a ten frame (a 2 by 5 grid), show the other part outside the ten frame, then fill the empty spots in the ten frame as you add to ten, then add the rest.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use make ten to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use make ten. Besides the routine described in the bullets above, sometimes do the following.

- Ask what addition problem they did and how it could be written as an equation.
- Ask students to create problems where make ten can be used.
- Discuss when make ten can be used efficiently, that is, when one of the parts is close to ten and the other makes the whole more than ten.
- Compare these problems to counting on and using double problems. Have the students choose whether they would count on, use doubles, or make ten to solve several different problems.
- After the students can make ten, a few minutes of practice on a regular basis for two or three weeks will enable them to solve make ten problems in about 3 seconds. This strategy will enable them to solve the remainder of the addition facts. They now have an efficient way to solve each of the 100 addition facts. With practice of these strategies, the students should easily be able to respond to any addition fact in less than 3 seconds.

Before the make ten strategy can be used successfully by students, they will need to know how to add ten and a single digit number. This is a prerequisite for the make ten strategy. The students should understand adding ten and a single digit number from their work with place value. If not, or as a review, use a ten frame or place value materials, such as base ten blocks, etc., to show how easy it is to add these numbers. Even some quick mental computation problems, such as, $10+6 ; 3+10 ; 40+7 ; 4+30$; etc, can help students recognize how place value ideas can help them solve these problems easily.

The students should also know how many more it takes to make ten when you start with 8 or 9. These are relatively easy since it only takes one or two more. This same thinking can be used with numbers other than 8 or 9 . But, students may not be as sure about how many more it will take to make ten and they may have more difficulty subtracting that number from the other part to determine how many extras need to be added to ten to get the whole.

Familiarity with the ten frame will help students learn these partitions of ten. Hide what you are doing. Put some counters in the ten frame. Uncover it for about 1 or 2 seconds and ask the students what number was shown. Also ask how many more it will take to make ten, that is, completely fill the ten frame. Repeat using different numbers. The students should quickly learn 6 is five and one more, 7 is five and two more, etc. It is a small step from that to know how many more it will take to make ten.

Other activities that can help students learn partitions of ten include the following. Start with 10 counters. Hide them and put some in one hand with the others in your other hand. Show how many counters there are in one hand. Ask how many there are in your other hand.

This strategy is a little more difficult for some children to learn, partly because of the prerequisites described above. But that does not make it less important. It just means that a few more activities need to be provided over an extended period of time.

Show some counters (less than ten). Ask how many more it will take to make ten. Have the students predict, then use counters to check. Repeat starting with other numbers. A similar question can be asked for larger numbers. For 37, ask how many more will it take to make 40. This is a question that can be asked just to help promote number sense.

It is easy to concretely illustrate the make ten strategy with the ten frame. Simply show an addition problem by putting 8 or 9 counters in the ten frame and some others outside the ten frame. The whole should be more than ten. Ask the students to show how they can use the ten frame to help them add the numbers. Then have them verbalize the thinking, for example, 8 and 2 more is 10 , then 4 more is 14 . Repeat with other numbers. Use this activity a few minutes each day for several days.

Another way to illustrate this thinking is to use a number line. For example, start at 8 , jump 2 to make 10, then jump 4 more to get the whole of 14 . If you draw the number line on the board, you can make it "numberless" except for the 8 and the 10, that is an open number line. As the students explain the make ten thinking, draw an arrow showing a jump of 2 to make ten, then draw another arrow showing a jump of 4 more to get to the whole of 14. Discuss how you added 6 more by adding 2 to make ten, then 4 more to make 14.

