How Can We Teach Thinking Strategies For Basic Subtraction Facts?

How does the part-part-whole language help students learn subtraction facts?

Children have much more difficulty learning subtraction facts than addition facts. This can be attributed to several things.

First, counting what is left to solve subtraction facts tends to put the focus on counting rather than on the relationship among the parts and the whole. By the time children have counted what is left, they are not thinking about how many there were in the whole or how many were taken away. The subtraction fact will not be learned until the three numbers, the parts and the whole, are related. Also, the subtraction generalizations and counting back do not help students with as many subtraction facts (only 37) as the addition generalization and counting on help with addition facts (64). Part of this difference is that counting up to subtract is more difficult because students do not understand the connection between counting up and taking way. Students also have much more difficulty counting back than they do counting up. Consequently, many students are not able to count back three easily. For these reasons, there are many more difficult subtraction facts than difficult addition facts.

Second, the language that students use for subtraction often is "take away." That language does not encourage students to think about how addition can help them. Students tend to think of subtraction as something entirely new that they have to learn.

Using part-part-whole language with subtraction facts can help students understand the relationship between addition and subtraction. The teacher should just naturally use that language for addition (the parts are 4 and 3, so the whole is 7). Then the teacher should continue using the same language for subtraction (the whole is 7 and one part is 3, so the other part is 4). The students will begin to understand they can use what they know about addition to help them with subtraction facts. This will help students identify number families, such as 3, 4, and 7, as number relationships that can help them solve either addition or subtraction fact problems.

The part-part-whole language also has some added benefits besides helping students learn subtraction facts. This language helps students become better problem solvers. Identifying the parts and the whole is helpful in many different problem-solving situations, but it is particularly helpful for word problems. Simply trying to represent a word problem as parts and the whole can help students decide which operation is appropriate to solve the problem. For example, if you know both parts, add to find the whole. If you know the whole and one part, subtract to find the other part.

The part-part-whole language even works for comparison problems. Comparison problems can be represented as one small set, one large set, and the difference between the two sets. The large set can be partitioned into the part that matches the small set and the difference. For comparison problems, if you know both parts of the large set, add to find the whole. If you know the whole and one part of the large set, subtract to find the other part, whether that be the part that matches the small set or the difference. A second grader explained this to the other students in one class where the part-part-whole language had been used informally by the teacher throughout the school year.

The part-part-whole language can also help students make sense out of the symbols that they write for addition and subtraction number sentences. The parts that are added are on either side of the plus sign. The whole is equal to the sum of these parts. For subtraction, the whole minus one part equals the other part. Students who have learned this connection to number sentences do not solve the problem 3 + ___ = 5 by writing an 8 in the blank. The structure of the number sentence does not detract from the parts and the whole.

The part-part-whole language also can help students identify relationships among many other numbers. If one fact is given, others can be derived from that, even for larger numbers. For example, students who have learned about parts and wholes, can solve problems like 35 - 16; 36 -16; 35 - 20; 16 + 18; etc, if they are provided the information that 19 + 16 = 35.

How can we break the pattern of take away and count what is left?

As children first begin learning to subtract, they typically will directly model the problem by showing the whole, counting to take away the number being subtracted, then counting what is left. This procedure is expected initially, but it is very inefficient for larger numbers and students and students tend to rely on counting instead of the parts and the whole. Consequently, this procedure does not promote learning the subtraction facts nearly as well as some of the more efficient thinking strategies. Students need to be encouraged to learn new ways of thinking.

Counting what is left, after you have taken a part from the whole, provides a way of solving subtraction problems that is comfortable for children. They already know how to count. And they are usually confident in their ability to count what is left to get the answer. Consequently, you need to create situations where the students can not easily count what is left. Teaching strategies to encourage new thinking are similar to those used to encourage students to use something other than count all for addition. In other words, hide the part that is left so the students can not count it and/or use larger numbers where counting is obviously not efficient.

Hiding the part that is left can be accomplished by showing the whole, covering it and removing a part, then asking how many are still covered. For example, ask how many bears there are. Show them all going into a cave (under your hand). Then have a part come out of the cave. Ask how many are still in the cave. Structure other situations where the part that is left is still hidden.

Counting what is left to subtract often continues to be an inefficient strategies that is used by students who have access to manipulatives too long. Some manipulative programs encourage teachers to let the students use these manipulatives. That is fine while the children are learning the concept of subtraction--what it is, how you represent it, and when you would use it. But, when you begin to want students to learn efficient ways to solve fact problems, the use of manipulatives should be structured so the students can not count what is left.

How can we teach counting back for subtraction?

Create a word problem where the part being subtracted is small (1 or 2).

- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they counted back. If not, then tell the students that you heard a student solve it by counting back, then explain how they were thinking. For 6 - 2, think 6, … then count back two, … 5, 4.
- You may want to model the counting back thinking by showing the whole, then covering it and reminding the students how many there are, then subtract the small part one at a time as you count back to get the whole.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use counting back to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use counting back. Besides the routine described in the bullets above, sometimes do the following.

- Ask what subtraction problem they did and how it could be written as an equation.
- Ask students to start at 8 and count back 2 and compare it to take away and count what's left. Reinforce the efficiency of counting back.
- Discuss when counting back can be used efficiently, that is, when one of the parts being subtracted is small.
- After the students can count back, a few minutes of practice on a regular basis for two or three weeks will enable them to solve counting back problems in about 3 seconds. If they also know the generalizations, they will be able to solve 37 of the 100 basic subtraction facts quickly.

Helping children learn to count back is just a step in the direction of helping them learn to use addition to help them solve subtraction facts. The most important teaching strategy is to hide the part that is left so the students cannot count what is left. This procedure can be used in situations where a small number is being subtracted.

Show a set of counters. Create a situation where they will be covered or hidden. Remove one or two of them, one at a time. Ask how many are still covered. By uncovering the counters one at a time, the students will be encouraged to count back to determine how many are still covered.

Create other situations where the part the is left after subtracting is hidden. Remove the counters one at a time. Perhaps redo the problem and have the students verbalize the counting back as you remove the counters one at a time.

How can we teach counting up for subtraction?

- Create a word problem where the part being subtracted is nearly as large as the whole (with a difference of 1 or 2).
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they counted up. If not, then tell the students that you heard a student solve it by counting up, then explain how they were thinking. For 8 - 6, think 6, … then count up to eight, …7, 8. That's two more, so the answer is 2.

- You may want to model the counting up thinking by showing the whole, then covering it and reminding the students how many there are, then subtract the part by removing all of them at the same time.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use counting up to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use counting up. Besides the routine described in the bullets above, sometimes do the following.

- Ask what subtraction problem they did and how it could be written as an equation.
- Ask students to start at 8 and count up 2 for the fact problem 10 - 8 and compare it to counting back 8 from 10. Reinforce the efficiency of counting up.
- Discuss when counting up can be used efficiently, that is, when the part being subtracted nearly as large as the whole.
- After the students can count up, a few minutes of practice on a regular basis for two or three weeks will enable them to solve counting up problems in about 3 seconds. If they also know the generalizations and can count back, they will be able to solve 51 of the 100 basic subtraction facts quickly.

Children who only think of subtraction as take away will have difficulty believing that you can count up to subtract. Take away and counting up are inconsistent in their minds. However, students who have learned about parts and wholes will understand that you just need to determine the other part. It does not matter whether you count up or count back. Another way that children can make sense of counting up as a reasonable way to subtract is to think of subtraction as a comparison situation where you are trying to find the difference between the numbers. Again, it does not matter whether you count back or count up to find that difference.

So, before you can expect success with this strategy, students should begin to understand that subtraction is more than just take away. There are two kinds of experiences that can help prepare students. The teacher can informally use the part-part-whole language as they work with addition and subtraction situations. The teachers can also present word problems and other problem solving situations that involve problem structures which are comparisons between numbers.

If helping children learn to count back is a step in the direction of helping them learn to use addition to help them solve subtraction facts, counting up is a leap in that direction. The most important teaching strategy is to hide the part that is left so the students cannot count what is left. This procedure can be used in situations where the number being subtracted is nearly as large as the whole.

Show a set of counters. Create a situation where they will be covered or hidden. Remove nearly all of the counters, all at the same time. Ask how many are still covered. If they can see nearly all of the whole, children often recognize that they only need one more or two more to make the whole. After a student suggests starting at the part and counting up to get the whole, try counting up as you show the hidden counters just to confirm that this strategy actually works.

Create other situations where the part that is left after subtracting is hidden. Remove nearly all the counters. Verify the answers by counting up as the hidden counters are shown. From this point, it is not a very large step to begin adding on one or two more to subtract.

How can we teach using doubles for subtraction?

- Create a word problem where the part being subtracted is about half as large as the whole.
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they used a double. If not, then tell the students that you heard a student solve it by using a double, then explain how they were thinking. For 13 - 6, think 6 and 6 make 12, so it must be 1 more or 7.
- You may want to model the use doubles thinking by showing the whole, then covering it and reminding the students how many there are, then subtract the part by removing all of them at the same time. Then ask if the hidden part could be the same as the part that is showing. Double that number to check. Adjust by one or two to make any needed corrections.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use doubles to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use doubles. Besides the routine described in the bullets above, sometimes do the following.

- Ask what subtraction problem they did and how it could be written as an equation.
- Ask students to start at 8 and count up to 15 for the fact problem 15 - 8 and compare it to using doubles. Reinforce the efficiency of using doubles.
- Discuss when using doubles can be used efficiently, that is, when the part being subtracted about half as large as the whole.
- After the students can use doubles, a few minutes of practice on a regular basis for two or three weeks will enable them to solve these problems in about 3 seconds. If they also know the generalizations and can count back and count up, they will be able to solve 70 of the 100 basic subtraction facts quickly.

Before you can expect success with this strategy, students should begin to understand that subtraction is more than just take away. There are two kinds of experiences that can help prepare students. The teacher can informally use the part-part-whole language as they work with addition and subtraction situations. The teachers can also present word problems and other problem solving situations that involve problem structures which are comparisons between numbers.

If helping children learn to count up is a leap in the direction of helping children learn to use addition to help them solve subtraction facts, using doubles is almost there. In fact, the strategy requires you to double the number you are subtracting, then adjust. You are adding. The double might not be the whole, and, if not, you have to go back and adjust the number you are adding.

The students also need to know some of the doubles before they will be able to use this strategy successfully. They can still learn the strategy by using known doubles, such as, 5 + 5 = 10, but they will not be able to use the strategy with many facts until they know the doubles well.

The most important teaching strategy is to hide the part that is left so the students cannot count what is left. This procedure can be used in situations where the number being subtracted is about half as large as the whole.

Show a set of counters. Create a situation where they will be covered or hidden. Remove about half of the counters, all at the same time. Ask how many are still covered. After a student suggests doubling the part, try it and adjust by one or two as needed. For 12 - 5, think 5 + 5 is 10, so you need to try a number that is 2 more than 5. The answer is 7.

Create other situations where the part that is left after subtracting is hidden. Remove nearly all the counters. Verify the answers by doubling the part that is subtracted. From this point, it is not a very large step to begin using known addition facts to help with subtraction facts.

How can we teach using ten for subtraction?

- Create a word problem where the whole is greater than ten and one of the parts is 8 or 9.
- Give the students an opportunity to think about this problem.
- Then ask several students to share how they figured the problem out. One of them probably will explain how they used ten. If not, then tell the students that you heard a student solve it by using ten, then explain how they were thinking. For 13 - 9, think 9 and 1 more is 10, and 3 more is 13. That's 4 more. Or, start at 13 and subtract. Think 13 minus 3 is 10, then subtract 1 more to get 9. That's 4 less, so the answer is 4.
- You may want to model the use ten thinking by showing the whole, then covering it and reminding the students how many there are, then start with 9 and add 1 to make ten, then 3 more to make 13. That's 4 more. You can also start at the 13 and subtract to ten, then subtract 1 more to get to 9.
- Verbalize the thinking and ask the class to verbalize the thinking.
- Then ask the class to use ten to solve another problem.

Using a similar procedure for a few minutes everyday for two or three weeks will help nearly all of the class learn to use ten. Besides the routine described in the bullets above, sometimes do the following.

Ask what subtraction problem they did and how it could be written as an equation.
Ask students to start at 8 and count up to 15 for the fact problem 15 - 8 and compare it to using ten. Reinforce the efficiency of using ten.
Discuss when using ten can be used efficiently, that is, when one of the parts being subtracted close to ten.
After the students can use ten, a few minutes of practice on a regular basis for two or three weeks will enable them to solve these problems in about 3 seconds. If they also know the generalizations and can count back, count up, using doubles, they will be able to solve 88 of the 100 basic subtraction facts quickly.

Using make ten for addition will help students learn to use ten for subtraction. It is also important for them to understand the part-part-whole concept and/or comparison situations so that adding from the part to the whole makes sense as a way to solve subtraction problems.

One way to help students learn this strategy is to provide subtraction situations where the whole is greater than ten and one of the parts is eight or nine. Then create a situation where the part that is left after subtracting is hidden. Then as a student is explaining the use ten strategy, you can put counters back with the hidden set as you add on. For example, 9 and 1 more is 10, then 4 more is 14.

Another way to illustrate this thinking is to use a ten frame. If the part being subtracted is close to ten, show it in the ten frame. Then add on as a student explains, 1 more is 10 and 4 more is 14.

The ten frame can also be used by showing the whole in the ten frame. Note that it will fill the ten frame and there will be some extras. Then subtract to ten and subtract the rest from ten as you remove counters from the ten frame.

The number line can also be used. Draw a number line on the board. For 14 - 9, just mark the numbers 9, 10, and 14 on the number line. Then draw arrows as you add to show 9 and 1 is 10, then 4 more is 14. That's 5 more. Or, subtract from 14 to 10, then subtract to 9.

How can we teach using addition facts for subtraction?

This strategy works efficiently for all subtraction facts. It helps children connect addition and subtraction ideas in a way that enables them to answer questions about either addition or subtraction if they know the answer to the other.

By informally using the part-part-whole language over an extended period of time, children will learn that known addition facts can be used to help students solve subtraction problems. Knowing the parts and the whole informs us about all of the addition and subtraction problems in the number family.

As described above in the other subtraction strategies, hiding the set that is left after subtracting encourages students to think in a way other than counting what is left. The strategies that evolve from these situations lead to using addition to help. The hidden set encourages students to use addition. The ten frame and the number line can be used to confirm that thinking and help children make sense of it.