

Suggestions For Using The Lessons At The K-2 Grade Levels

Background Information

Before you start the lessons, it is important to know that you will not start with lesson 1 and use one after the other until you have completed all the lessons. Altogether there are about 850 5-minute lessons, ...far more than what you will have time to use and far more than what you will need.

By listening to the students explain their solutions, you will know when they have developed understanding and the confidence to move on to the next group of lessons. You can always revisit these lessons as review or as needed. Each group of lessons has a 5-item assessment to inform and/or confirm your decisions.

Representations

Chapters 1 and 9 introduce the ten frame and the number line. These representations, together with the animations used with them, are used to help students visualize and make sense of addition and subtraction. Familiarity with these representations will help students prepare for the addition and subtraction lessons.

Addition And Subtraction

Chapters 2-8 and 10 are explore lessons and previews for what students will master later in their regular school mathematics curriculum. Daily use of Thinking With Numbers provides extra time for students to explore and preview addition and subtraction concepts in an informal, low-stakes environment. There is not one lesson in chapters 2-8 and 10 that needs to be mastered before moving on. But collectively over several months, they will help students make sense of addition and subtraction.

Reasoning Strategies

Chapters 11 and 12 provide structured lessons to help students make sense of reasoning strategies for addition and subtraction. After students can spontaneously use each strategy, there are practice the strategy lessons to help develop reasonable fluency in using that strategy.

Why Should These Lessons Be Used In September When You Don't Teach That Topic Until February?

Exploring a topic for just a few minutes a day over an extended period of time has proven to be much more effective than spending more time per day on fewer lessons or even a unit of instruction. The research on this has been definitive for at least 75 years. Originally, the focus was on massed versus distributed practice and the focus was on skills. But recent evidence supports the idea that meaningful distributed instruction also works extremely well for conceptual learning. The focus of these lessons is on conceptual learning.

Making sense of addition and subtraction involves far more than "getting the answer." By exploring these concepts over time, they will become familiar and comfortable:

- with ways to represent these concepts,

- with actions on models such as ten frames and number lines,
- using appropriate language,
- with appropriate symbols,
- with different ways of reasoning, and
- knowing when to apply these ideas in everyday life.

Daily use of these lessons will provide the experiences and time students need to deeply understand all of these aspects of addition and subtraction and begin to develop flexibility in their use. Previewing these topics (chapters 2-8 and 10) provides that opportunity.

Outline Of The Lessons

Getting Ready For Addition

- Chapter 1 Ten Frame (30 lessons and 3 assessments)
- Chapter 2 Making Sense Of Addition
 - Part 1 Joining: How Many In All? (36 lessons and 6 assessments)
 - Part 2 Describing: How Many In All? (36 lessons and 6 assessments)
- Chapter 3 Equals With Addition (10 lessons and 1 assessment)
- Chapter 4 Parts Of Numbers (30 lessons and 3 assessments)

Getting Ready For Subtraction

- Chapter 5 Making Sense Of Subtraction
 - Part 1 Separating: How Many Are Hidden? (36 lessons and 6 assessments)
 - Part 2 Describing: What Is The Other Part? (36 lessons and 6 assessments)
- Chapter 6 Equals With Subtraction (10 lessons and 3 assessments)
- Chapter 7 Equalizing (10 lessons and assessments)

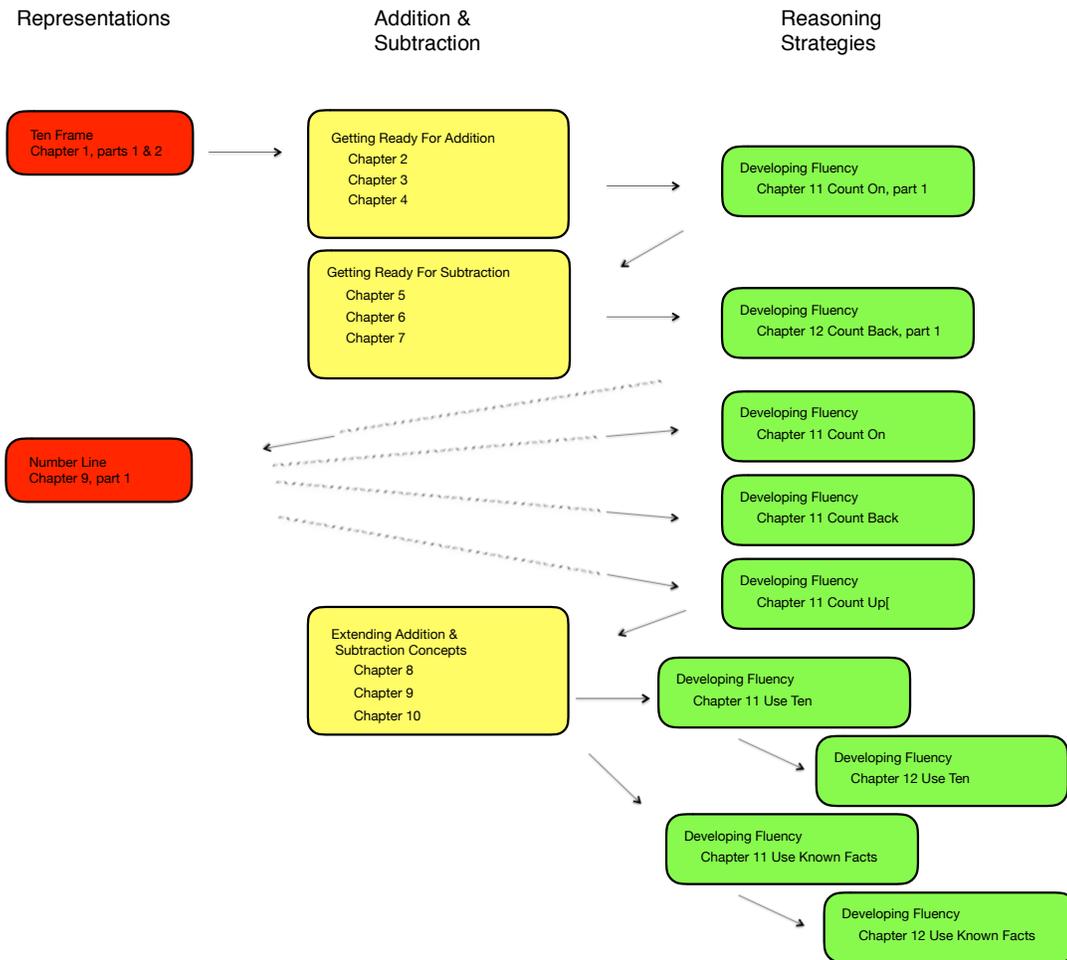
Extending Addition And Subtraction Concepts

- Chapter 8 Relating Addition And Subtraction
 - Part 1 Joining: How Many More? (36 lessons and 6 assessments)
 - Part 2 Joining: What Did You Start With? (36 lessons and 6 assessments)
 - Part 3 Separating: What Did You Start With? (36 lessons and 6 assessments)
 - Part 4 Separating: How Many Moved? (36 lessons and 6 assessments)
- Chapter 9 Number Line (12 lessons and 2 assessments)
- Chapter 10 Making Sense Of Comparison
 - Part 1 How Many More Or Less? (36 lessons and 6 assessments)
 - Part 2 How Many In The Big Set? (36 lessons and 6 assessments)
 - Part 3 How Many In the Small Set? (36 lessons and 6 assessments)

Developing Fluency

- Chapter 11 Reasoning With Addition
 - Counting On To Add (38 lessons and an on-line assessment with reports)
 - Using Ten To Add (38 lessons and an on-line assessment with reports)
 - Using Known Facts To Add (43 lessons and an on-line assessment with reports)
 - Flexible Use Of Strategies
 - Tree Diagram (25 lessons)
 - Open Number Line (25 lessons)
- Chapter 12 Reasoning With Subtraction
 - Counting Back To Subtract (38 lessons and an on-line assessment with reports)
 - Counting Up To Subtract (38 lessons and an on-line assessment with reports)
 - Using Ten To Subtract (38 lessons and an on-line assessment with reports)
 - Using Known Facts To Subtract (43 lessons and an on-line assessment with reports)
 - Flexible Use Of Strategies
 - Use Nice Numbers, Then Adjust (15 lessons)
 - Change The Problem (10 lessons)
- Chapter 13 Choose A Strategy (10 lessons)

Flow Chart For Lessons



Comments About The Flow Chart

There is no right or wrong way to select which lessons to use any of the explore lessons in Chapters 2-8 and 10. They can be used at any time as long as the students do not get frustrated with the level of difficulty. Furthermore, most students will not need to do every lesson in each chapter. By listening to student explanations and using the paper-and-pencil formative assessments at the end of each section of the lessons, you will be able to identify an appropriate level of difficulty and know when to move on.

The lessons on the ten frame, Chapter 1-parts 1 & 2 need to be used prior to using Getting Ready For Addition-chapters 2-4. After students can comfortably and confidently answer the questions in chapters 2-4, Developing Fluency-Chapter 11-Counting On-part 1 can be used. The remainder of Chapter 11-Counting On can be used after Chapter 9-Number Line-part 1 has been introduced. Note that fluency is never mentioned and not expected until Chapter 11-Counting On-Practice Counting On-part 4, is being used. It is crucial that

students understand before trying to develop fluency. Delay expectations for fluency until they do understand.

Similarly, the lessons on the ten frame, Chapter 1-parts 1 & 2 are prerequisites for using Getting Ready For Subtraction-chapters 5-7. After students can comfortably and confidently answer the questions in chapters 5-7, Developing Fluency-Chapter 12-Counting Back-part 1, can be used. The remainder of Chapter 12-Counting Back can be used after Chapter 9, Number Line, part 1, has been introduced. Note that fluency is not expected until Chapter 12-Counting Back-Practice Counting Back-part 4 is being used.

Then Developing Fluency, Chapter 12, Counting Up can be used. Again, fluency is not expected until Chapter 12-Counting Up-Practice Counting Up-part 4 is being used.

At this point, students should be reasonably fluent and confident in using counting on to add and counting back and counting up to subtract. By practicing the strategies, the students gradually will be able to use conceptual thinking more quickly. There is no absolute standard for response time expected. Students are measured against their own prior performance in the on-line assessments.

Next, the lessons in Extending Addition And Subtraction-chapters 8-10 can be used. The structure and language of these problems is often more difficult for students, so they may need more experiences and time to understand.

After students are comfortable and confident in answering the problems in Chapter 8, they may be introduced to the strategy lessons using ten and using known facts. Since there are four strategies, two for addition and two for subtraction, it probably makes sense to use each addition strategy before using the corresponding strategy for subtraction. After students are capable of confidently using each of these strategies, part 3 of the lessons for each strategy, can be used to gradually develop fluency.

Comparison lessons, Chapter 12, can then be used to further extend addition and subtraction concepts. Students may need more experiences because comparison problem structures are more complex and the language is more difficult for students to understand (How many more? How many fewer? How much more? How much less?).

Chapter 13 provides students opportunities to compare the use and efficiency of different strategies. There is no right or wrong way to solve these problems. Students will each have their own idiosyncratic way of reasoning.

Making A Plan

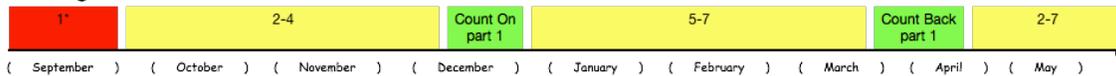
To organize the lessons, schools should identify when each reasoning strategy will be taught in their curriculum and when they expect fluency with that strategy. Note that fluency does not mean an instantaneous response. It means a student is able to start the thinking process for a strategy and the ideas flow smoothly as the student uses that strategy. There should be no hesitation while the student needs to recreate what to do next. The

time it takes a student to complete this reasoning process is highly individual, so no absolute time demands are appropriate. After practice using each strategy, most students are able to think through these reasoning processes in 2 or 3 seconds.

The following is one possibility. Remember that it all depends on when your school curriculum teaches the strategies and when the school expects fluency.

One Sample Plan

Kindergarten



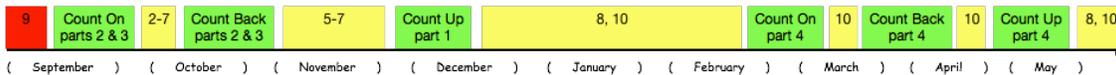
* The numbers in the colored boxes above indicate chapters.

Suppose you decide to introduce the students to counting on in December and counting back in April. Explore and preview lessons can be used at all other times. Start with an introduction to Chapter 1, the ten frame. Then chapters 2-4 can be used prior to introducing counting on. Chapters 5-7 can be used after counting on and prior to introducing counting back. Chapters 2-7 can be revisited for the remainder of the school year.

At the end of kindergarten, students will have :

- made sense of the ten frame,
- explored joining and separating,
- explored parts and the whole,
- been introduced to counting on and counting back as ways to solve addition and subtraction problems, and
- an excellent preparation for learning addition and subtraction in grade 1.

Grade 1

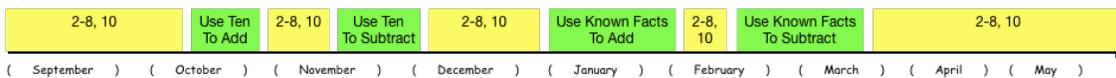


Suppose you decide to continue developing counting on and counting back with the number line in the fall, then introduce counting up to subtract in December. Suppose also, that practice to develop better fluency with these strategies is presented in the spring. Explore and preview lessons can be used at all other times. Chapters 2-7 can be revisited in the early fall. Chapters 5-7 can be revisited prior to introducing counting up to subtract. Chapters 8 and 10 can be explored and previewed before and between practice with these counting strategies.

At the end of grade one, students will have :

- made sense of the number line,
- continued their development of counting on and counting back,
- been introduced to counting up to subtract,
- explored relating and extending addition and subtraction concepts,
- practiced counting strategies to develop fluency, and
- an excellent preparation for mastering addition and subtraction in grade 2.

Grade 2



Suppose you decide to introduce the students to using ten in the fall and using known facts in January. Explore and preview lessons from chapters 2-8 and 10 can be used at all other times.

At the end of grade 2, students will have :

- made sense of the open number line and tree diagram
- made sense of all addition and subtraction problem structures,
- made sense of counting and derived fact strategies as ways to solve addition and subtraction problems,
- developed flexibility in the use of reasoning strategies,
- learned when to use these concepts in everyday life,
- developed reasonable fluency with all basic facts, and
- an excellent preparation for learning addition and subtraction in grade 3.