What Are The Thinking Strategies?

What are some important thinking strategies for addition?

Initially, children count all. This means that they count or show both parts of the problem and then proceed to count them all from one to the total. This is only efficient if both of the parts being added are very small. This is an appropriate method for solving addition problems initially, but children should be encouraged to learn more efficient ways of solving problems with larger numbers. Students soon learn that adding zero is easy. $n + 0 = n$ is a generalization that enables them to solve problems they have never seen before. They just know that adding zero does not change the other part.

As children progress from counting all, they begin to count on. This means they start with one part and count on by ones to get the total. They soon learn to use the commutative property and start with the larger part before they count on the smaller number. Generally, children will not need to count on more than three, but some children can do it easily. For a problem like 3 + 8, the students just think 9, 10, 11.

Students can then learn to use a fact that they already know to help them figure out facts they do not yet know. These are called derived fact strategies. For example, counting on will soon help children memorize some facts. If they know that 7 + 3 is 10, it is not too difficult to understand that 7 + 4 is just one more.

One derived fact strategy to help students learn some of the larger facts, is to use doubles to help them with near doubles. For example, to solve 6 + 7, many students will think 6 + 6 = 12, so 6 + 7 is just 1 more or 13. Students soon are using this with problems where the parts have a difference of one or two. In fact, some students learn to use the "Robin Hood" method for numbers with a difference of two. For 8 + 6, you can take one from the 8 and put it with the 6 to make 7 + 7. Similar thinking can be used with any fact that a student knows. Many children use this thinking with ten. For example, for 9 + 6, since 10 and 6 is 16, 9 and 6 is just one less.

Another derived fact strategy involves making ten. When one of the parts is nearly 10, you can just add on to get 10, then add the extras. For example, for 9 + 5, you can think 9 and 1 more is 10, then 4 more is 14. This thinking is a powerful strategy to use with mental computation with larger numbers. It also is very helpful when students are beginning to learn the hard multiplication facts. For example, for 3 x 9, you can think 2 nines is 18, now just add on another nine, 18 and 2 more is 20, then 7 more is 27.

Which basic facts can efficiently be solved by these addition strategies?

The zero generalization can be used to help students learn any basic fact where one of the parts is 0. That gives the students a quick way to solve 19 basic facts.

If students count on 1, 2, or 3 more, they can quickly solve basic facts where one of the parts is a 1, 2, or a 3. That gives the students a quick way to solve 45 additional basic fact problems.

Learning the doubles and using them to solve near double problems enables students to easily solve another 24 problems. Consequently, using the zero generalization, counting on, and using doubles can help children figure out any of 88 facts very quickly. If they practice using these strategies to speed up their thinking, they can respond to any of these facts in less than 3 seconds.

Learning to make ten gives the students an efficient way to solve most of the remaining basic addition facts. However, since there are only a few basic facts left, the real power of this strategy is to help students make sense of adding larger numbers, that is mental math, and to help them learn multiplication facts by starting with a known fact and adding on one more group. For example, 7 x 4 is 28, so 8 x 4 is just 4 more. 28 and 2 more is 30, then 2 more is 32.

What are some important thinking strategies for subtraction?

Children initially show the whole, take away a part, and count what is left to get the answer. This is commonly called take-away and count what is left or count all. Initially, this strategy is appropriate, but children should be encouraged to learn more efficient thinking for larger numbers.

There are two zero generalizations, that students learn about subtraction. One involves subtracting zero, n - 0 = n. The other involves subtracting a number from itself, n - n = 0.

After counting all, students learn some other counting strategies. If students are subtracting a small number, counting back is efficient. For example, for 9 - 2, you can just think, 8, 7. Counting back is not as easy as counting on, but most students can count back 1 or 2. Some students count back more, but this strategy becomes inefficient to count back many more than two.

Similarly, if the part you are subtracting is nearly the same size as the whole, counting up is efficient. For example, for 9 - 7, you can just start at 7 and count up to 9. By keeping track of how many counts you make, you know the difference, ...8, 9. That's 2 more. This strategy can be easily used for problems where the difference is no more than 2. Some children use it for problems with greater differences, but again, it becomes inefficient for greater differences.

Sometimes students use doubles. For example, for 11 - 6, you can think, since 5 and 5 make 10, the other part must be 1 more or 6. This can be used as long as the part is about half of the whole. The other part can be found by adjusting or compensating after checking to see if doubling the part makes the whole.

Similarly, some students use ten to help them solve subtraction facts with the whole greater than 10. Ten is used as a bridge or stepping stone to get from the known part to the whole or from the whole to the known part. For example, for 13 - 9, you can start at 9 and add 1 to make ten, then add 3 more to get 13. That's 4 more. Or you can think, start at 13 and subtract 3 to get to 10, then subtract 1 more to get to 9. That's 4 less.

If the part being subtracted is just a little more than the one's digit of the whole, you can use ten by subtracting that amount and using ten as a stepping stone. For 14 - 6, think 14 minus 4 is ten, then subtract 2 more to get to 8.

Ten can also be used in a other ways. For example, for 13 - 9, you can think 13 - 10 is 3, so 13 - 9 is 1 more or 4. This is particularly effective when you are subtracting nine. Also, for problems with a difference of 8 or 9, you can use ten as a bridge or stepping-stone when you take away the part. For example, for 16 -7, 16 - 6 is 10, then take away 1 more to get 9.

All of these strategies are overridden by the one main strategy that you hope students will eventually begin to use. That is using addition facts that are already known. Since students know addition facts and each addition fact simply tells you the parts and the whole, they can use that information to answer related subtraction facts. For example, if you know that 3 + 5 is 8, then you also know 8 - 3 and 8 - 5 since the parts and the whole are the same for all of these basic fact problems.

Which basic facts can efficiently be solved by these subtraction strategies?

The zero generalizations allow students to quickly solve 19 of the basic subtraction facts. Subtracting 0 or subtracting a number from itself become easy.

Counting back can be used to quickly solve problems where you are subtracting 1 or 2. This includes 18 additional problems.

Counting up can be used to quickly solve problems where there is a difference of 1 or 2. This includes 14 additional problems. Together, the generalizations, together with counting back and counting up, provide students an efficient way to solve 51 of the basic subtraction facts.

Using doubles can provide a quick way to solve another 19 basic facts. In each of these cases, doubling the part is no more than a difference of one from the whole.

Using ten as a bridge or stepping stone can help students quickly subtract 8 or 9. Facts, with a difference of 8 or 9, can also be solved by using ten. This enables students to quickly subtract another 18 basic facts. Collectively, these strategies help children easily solve 88 of the 100 basic subtraction facts.

All other facts can all be efficiently solved by using know addition facts. In fact, as soon as students know addition facts and understand the part-part-whole relationship, they are able to solve subtraction by using what they already know about addition.