

How can you help your students learn and remember math?

Students often can answer a problem one day, but forget the next day, or a week later. It is most likely that you notice this with basic facts. While not as well documented, it also happens with reasoning strategies.

Reasoning strategies are important because achievement is closely related. Unlike memorizing a basic fact, each new strategy that students begin to use is accompanied by an increase in achievement.

What can you do to help students learn and remember these thinking skills and use them flexibly and fluently in everyday life?



First, what does not work?

William Brownell, "The Effects Of Premature Drill In Third Grade Arithmetic," 1935, found that drill for an extended amount of time (1) did not promote what we now call number sense for any student and (2) there were no long-term effects in promoting memorization or achievement for about 40% of all students. Results of more recent research have confirmed these findings and added the following.

Teaching practices that have no positive long-term effects include:

- using drill and timed tests to memorize basic facts
- telling students the answer or how to solve a problem, before giving them time to figure it out and discuss solution strategies
- not giving students time to explore with manipulatives, so they can develop number relationships
- giving them extended time to explore with manipulatives, without also holding students accountable to try to make sense of reasoning strategies beyond counting
- organizing your curriculum in units that do not provide enough time for students to make sense and develop flexibility and fluency



Then, what does work?

Teaching practices that have positive long-term effects on learning and memory include:

1. using meaningful instruction
2. making sense of multiple representations of concepts
3. making sense of multiple reasoning strategies
4. using a problem solving approach
5. using distributed instruction for an extended time
6. using frequent and brief low stakes ungraded assessments



1. Using Meaningful Instruction

Learning must be connected to a student's prior knowledge or it will not be used and soon be forgotten. Using objects to illustrate representations, the actions and the reasoning strategies to solve problems, together with verbal explanations, are far more effective than just symbolic instruction. These experiences, together with having students explain, compare, contrast, and evaluate solution strategies, lead to increased achievement.

Students, who can use addition and subtraction flexibly and fluently in everyday life, have made sense of the concepts:

- number concepts and relationships
- what it means to add and subtract, how they are related through parts & the whole, and when can you use them
- how to represent numbers, addition, and subtraction in a variety of ways
- a variety of reasoning strategies that can be used efficiently, depending on the context



2. Making Sense of Multiple Representations of Concepts

Knowing when a concept can be used in everyday life means recognizing similarities and relationships between the context and the representations students understand about the concept.

For example, students who understand subtraction only as take away, will not recognize they can use their knowledge about subtraction in a comparison situation. To them comparing the size of two sets is not take away. Similarly, without a knowledge of relationships among parts & the whole, students will not recognize they can use their knowledge about addition to help with subtraction.

Learning to represent concepts in a variety of ways enhances the ability of students to recognize opportunities when they can use their knowledge about that concept.



3. Making Sense of Multiple Reasoning Strategies

Young children begin solving addition and subtraction problems by counting. That works and is efficient for small numbers. Other strategies, such as using known facts or using ten, are far more efficient for larger numbers.

For example, to add $9 + 5$, students can think "One more is ten, then four more is 14." Or to subtract $14 - 9$, students can think, "One more is ten, and four more is 14. That's five more."

By comparing and contrasting counting, using ten, using known facts, and other strategies, students can learn to choose an efficient strategy for the context and the numbers involved. To use addition and subtraction flexibly and fluently, students need to develop multiple reasoning strategies and a way to choose which one might be efficient.



4. Using A Problem Solving Approach

Telling a student the answer or how to solve a problem does not help when the student confronts another problem. Students need time to solve a problem, discuss their solutions, come to a consensus on the answer, and sometimes consider the efficiency of their solution process. They need to look for relationships, both within and between problems. They need to ask, what if questions. They need to create their own problems. They need to develop confidence in their abilities to figure it out. They need repeated opportunities to experience this process.

This problem solving approach to teaching not only helps students become better problem solvers. It helps students develop confidence, persevere, and believe they can be successful with math.



4. Using A Problem Solving Approach

Students need to:

- solve problems, not be told how to solve them,
- share and explain their solution process,
- listen to other solution processes,
- compare, contrast, and discuss different solution strategies,
- come to a consensus about each answer, is it correct or not,
- reflect on the efficiency of different solution strategies, and
- create and solve similar problems.



5. Use Distributed Instruction For An Extended Time

Learning concepts and reasoning strategies so students can use them flexibly and fluently takes much more time than typical textbooks provide.

Rathmell found that it takes much longer than typical textbooks provide for students to make sense and begin to spontaneously use a new reasoning strategy. Despite being able to explain the thinking when asked, only about half of grade three students were spontaneously using ten to add after two weeks of ten-minute lessons. Even fewer were spontaneously using ten to subtract. When presented a problem where using ten would be helpful, it did not occur to them that they could use that thinking.

Students are not expected to master any of these lessons before moving to the next lesson. By presenting daily opportunities for an extended time, each student will make sense on their own time line.



6. Using Frequent, Brief, Low-Stakes Assessments

Having students take frequent low-stakes, ungraded practice quizzes, with appropriate feedback, helps with long-term retention. Each quiz provides another opportunity for students to compare and contrast different types of problems and to review concepts and reasoning strategies.

Just like distributed instruction (spaced learning) is more effective than blocks of instruction, frequent & brief quizzes are more effective.

