THINKING WITH NUMBERS

Lesson Descriptions

Making Sense Of Counting

Counting is much more than just memorizing numbers in order. Each of the common errors that children make is addressed by these lessons. They are presented problems involving one-to-one matching and how it can be used to identify the number of a set and how one-to-one matching can be used to compare sets. Children will learn how sets can be manipulated in ways that do not change the number and in ways that do change the number. Additionally, children will learn numbers that are 1 or 2 more and 1 or 2 less than a given number. Making sense of counting and matching and how they can be used to compare sets will provide an understanding that will help children be successful with numbers in school math.

Expected content outcomes include helping children learn:

- the number names and symbols,
- what number comes next,
- that the last number you count tells how many,
- that the order you count objects does not change the number,
- how to match one-to-one,
- that matching sets can help you compare,
- two sets can be compared indirectly by matching each to a third set,
- that moving objects does not change the number,
- that replacing objects does not change the number,
- that adding or removing objects does change the number,
- numbers that are 1 or 2 more, or 1 or 2 less,
- to accurately count a set of objects and tell how many, and
- to accurately tell if the number has changed by 1 or 2 after a set has been manipulated (objects have been moved or replaced; objects have been added or removed).

THINKING WITH NUMBERS

Lesson Descriptions

What Comes Next?

Children at a very young age learn that the order in which you say the number names is important. They know there is a constant order and often will consistently repeat an incorrect sequence. Every child learns our number sequence gradually. It is not uncommon for a young child to count "1, 2, 3, 6, 4." It is common for that child to say the same numbers in the same order until they learn that 4 comes after 3. These lessons will help children learn what number comes next and begin to recognize that the last number you say tells how many.

Expected content outcomes include helping children learn:

- what number comes next in the sequence,
- that the last number you say tells how many, and
- to accurately count a set of objects and tell how many, and
- to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

One-To-One Matching

When young children begin to count, they often say the counting numbers in the right sequence, but do not coordinate pointing at objects with the counting sequence. Sometimes they point at an object without saying a number name; sometimes they skip an object; sometimes they count an object more than once; sometimes they say extra number names. They need to learn to match a number name with each object once and only once, that is, they need to make sense of one-to-one matching.

Expected content outcomes include helping children learn:

- to count each object only once without skipping any objects,
- to eliminate counting errors,
- to accurately count a set of objects and tell how many, and
- to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

How Many Dots Are There?

Even after counting the objects in a set correctly, young children often do not know the number of objects. These lessons will help children learn that the last number you say tells how many. It is not uncommon for a young child to count objects, "1, 2, 3, 4." Then, when asked how many, the child might respond "7." By hiding the objects under a cloud, they have to rely on the last number counted to tell how many. They need to begin feeling comfortable using their prior knowledge about the number of a set without having to recount each time they are asked how many.

Expected content outcomes include helping children learn:

- that the last number you say tells how many,
- to accurately count a set of objects and tell how many, and
- to recognize the symbols for the numbers.

Order Of Counting

Young children do not know that the order you count a set of objects does not change the number. For example, if you count left to right, you get the same number as if you count right to left or in any other order. By having children compare the results of counting left to right with counting right to left, they will learn that the order in which you count does not change the number.

Expected content outcomes include helping children learn:
- that the order in which you count objects does not change the number,
- to accurately count a set of objects and tell how many, and
- to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

Matching Sets

Children often begin to count when asked how many. They don't think about prior knowledge they may know to help answer that question. If there are six people seated in a room, do you have to count to tell how many chairs are being used? If they know the number of one set and that set matches a second set, they also know the number of the second set, without counting.

Expected content outcomes include helping children learn:

- to recognize when two sets match,
- that two sets that match have the same number,
- to accurately count a set of objects and tell how many, and
- to recognize the symbols for the numbers.

Comparing Sets

When asked to compare sets, children often resort to counting. They may not need to count each time. When one set matches part of another set, they can tell which has more objects and which has fewer objects without counting. If you put a rose in each vase and there is an extra vase, are there more roses or vases? Do you have to count to know? By looking at the extras, children can tell how many more or how many fewer without counting both sets.

Expected content outcomes include helping children learn:

- to compare two sets by matching one to part of another,
- to tell that one set is 1 or 2 more and that one set is 1 or 2 fewer just by matching, without counting or necessarily even knowing the number of each set,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

Comparing Sets Indirectly

Two sets that are not seen at the same time can be compared if they are each matched to a third set, without counting. When there is an extra chair for the people in one room and those same people go to another room with not enough chairs, do you have to count to compare the chairs in one room with the other? When both are more than the third set or when both are less than the third set, then it is more difficult but not impossible.

Expected content outcomes include helping children learn:

- to compare two sets that are not seen at the same time by matching each to a third set,
- to tell that one set is 1 or 2 more and that one set is 1 or 2 fewer just by matching to a third set, without counting or necessarily even knowing the number of each set,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

Moving Dots

Even after counting objects, children often believe there are more objects when you spread the objects out (some believe there are fewer objects because they are spread more thinly). They need to learn that the number of a set is constant, even after the objects have been moved. Confronting children with this issue may help them resolve differences between what they believe and what they have counted. You may have to revisit these lessons over an extended period of time.

Expected content outcomes include helping children learn:

- that rearranging objects does not change the number of objects,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

Replacing Dots

Replacing objects, in a one-to-one match, does not change the number of a set. Children often will want to recount to check how many are in a group after you substitute two children for two already in the group. Rearranging objects or replacing objects, one-to-one, does not change the number.

Expected content outcomes include helping children learn:

- that replacing objects, one-to-one, does not change the number of objects,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

Adding And Removing Dots

Adding objects or removing objects does change the number of objects in a set. When you add objects, there are more. When you remove objects, there are fewer.

Expected content outcomes include helping children learn:

- that adding objects increases the number,
- that removing objects decreases the number,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

Changing The Number

These lessons will help children understand the relationship between to numbers that are close together. When one object is added, there is 1 more. When one object is removed, there is 1 fewer. By hiding these objects, children learn to rely on prior knowledge, without having to count each time.

Expected content outcomes include helping children learn:

- that adding an object increases the number by one,
- that removing an object decreases the number by one,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

One Or Two More

After counting a set of objects to tell how many, the objects are hidden and one or two more are added. Children need to learn that one more is just the next number and two more is just the number after the next number. By hiding the objects, children have to rely on prior knowledge, without counting each time.

Expected content outcomes include helping children learn:

- one more than a number is just the next number,
- two more than a number is just the number after the next number,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

One Or Two Less

After counting a set of objects to tell how many, the objects are hidden and one or two more are removed. Children need to learn that one less is just the number before and two less is just the number before the number before. By hiding the objects, children have to rely on prior knowledge, without counting each time.

Expected content outcomes include helping children learn:

- one less than a number is just the number before,
- two less than a number is just the number before the number before,
- to accurately count a set of objects and tell how many, and
- to begin to recognize the symbols for the numbers.

THINKING WITH NUMBERS

Lesson Descriptions

Making Sense Of Numbers & Partitions

Making sense of numbers involves a complex network of concepts and relationships. Children develop this knowledge and understanding gradually as they experience counting objects, joining sets of objects, separating sets of objects, recognizing number patterns and their relationships, and thinking about parts and the whole. For example, when there are 4 dots in the five frame, there is 1 empty box; 4 and 1 make 5. Similarly, if there is 1 empty box, there are 4 dots. Children know that without counting. Representing numbers by using the five frame (ten frame for numbers greater than five) provides a structure that enables children to recognize and identify numbers and their relationships with other numbers without counting. The animations involved, while putting numbers together and taking them apart, help children make sense of partitions of the numbers. Hiding objects under clouds as these actions occur encourages children to use the structure of the five frame, prior knowledge, and to develop new ways of thinking to solve problems.

Expected content outcomes include helping children learn:

- to recognize the number of objects in a five frame (ten frame for numbers greater than five) without counting,
- to recognize objects in common number patterns without counting,
- to take numbers apart and put them together without counting, and
- to develop flexibility and fluency in their thinking about numbers and their parts.

THINKING WITH NUMBERS

Lesson Descriptions

The Five Frame

These lessons encourage children to use the five frame to recognize relationships among numbers to five. The number of empty boxes provides as much information as the number of filled boxes. The number of filled boxes and the number of empty boxes is always 5. One more and one less relationships also become evident. It just takes 1 more than 3 to make 4. Or if you take 1 away from 4, there are 3.

Expected content outcomes include helping children learn:

- to recognize numbers without counting,
- to compare and relate numbers to five,
- put parts together to make a whole, and
- to decompose the whole into parts.

THINKING WITH NUMBERS

Lesson Descriptions

Number Patterns

These lessons will help children recognize common number patterns without having to count each time. For example, the domino pattern for five can easily be recognized without counting. By looking for parts of this configuration, children will recognize that there are 4 corner dots and 1 in the middle (4 and 1 make 5), there are 3 dots on a diagonal and 2 other dots on either side, (3 and 2 make 5). These patterns can help children make sense of the partitions of numbers.

Expected content outcomes include helping children learn:

- to recognize numbers in common patterns without counting,
- to recognize parts within the pattern, and
- to put parts together to make a whole, and
- to decompose the whole into parts.

Make A Number

Children will add on to a number to make a given number. The five frame provides structure to help children know how many empty boxes there should be and how many more are needed. The joining action will begin to help children make sense of addition and the plus sign. If you have 3 and want to make 4, 1 more is needed. There will be 1 empty box to make 4. One more than 3 will leave 1 empty box, so 3 and 1 make 4. Joining 3 and 1 can be represented by using the parts and plus sign; 3 + 1. The equals sign will be introduced later. The part-part-whole diagram will reinforce the partitions of numbers.

Expected content outcomes include helping children learn:

- to put parts together to make a whole,
- to recognize parts within the whole,
- to represent joining by using addition, and
- to use numbers and the plus sign to represent addition.

THINKING WITH NUMBERS

Lesson Descriptions

Take A Number Apart

When a part is removed from the whole, children will be asked to figure out the hidden part. For example, 4 dots are shown, then hidden. After 1 comes out of the cloud, how many are still hidden? The separating action will begin to help children make sense of subtraction and the minus sign. Separating 1 from 4 can be represented by using the whole, the separated part, and minus sign; 4 - 1. The equals sign will be introduced later. The part-part-whole diagram will reinforce the partitions of numbers.

Expected content outcomes include helping children learn:

- to take the whole apart and figure out the hidden part,
- to recognize parts within the whole,
- to represent separating by using subtraction, and
- to use numbers and the minus sign to represent subtraction.

THINKING WITH NUMBERS

Lesson Descriptions

Make Part Of A Number

These lessons show a number, hide it under a cloud, then ask children to figure out how many need to be removed to leave a given number. For example, if you start with 3 dots, then hide them, how many dots need to be removed to leave 1 dot under the cloud. Starting with 3, then removing 2, can be represented with numbers and the minus sign; 3 – 2. The separating action will begin to help children make sense of subtraction and the minus sign. The equals sign will be introduced later. The part-part-whole diagram will reinforce the partitions of numbers.

Expected content outcomes include helping children learn:

- to take part from the whole to leave a given number,
- to recognize parts within the whole,
- to represent separating by using subtraction, and
- to use numbers and the minus sign to represent subtraction.

THINKING WITH NUMBERS

Lesson Descriptions

More Parts

These lessons involve children with parts and the whole where there are more than two parts. The whole is the sum of its parts, whether there are two parts or more than two parts. After a number is shown, hidden, then two parts are shown, children will be asked to figure out the remaining part that is still hidden. For example, if 4 dots are shown, hidden, then parts consisting of 1 and 2 are shown, how many dots are still hidden.

Expected content outcomes include helping children learn:

- to figure out the missing part of a whole, after other parts are shown,
- to flexibly decompose numbers into parts, and
- to represent the parts using addition.

THINKING WITH NUMBERS

Lesson Descriptions

Making Sense Of Adding & Subtracting

Making sense of adding and subtracting involves far more than counting to get the answer. Learning should focus on two big ideas. First, we need to help children understand addition and subtraction. They need to make sense of how the parts and the whole are related—part + part = whole and whole – one part = other part. These relationships occur in many different everyday situations. Exploring With Word Problems will enable children recognize these situations as saddition and subtraction. The concrete representations and symbolization enable them to model these situations. Second, to become efficient, flexible, and fluent in using addition and subtraction, children need to make sense of a variety of reasoning strategies to enable them to efficiently answer addition and subtraction problems. Animations with the representations model the new thinking strategies and help them develop flexibility and fluency.

Expected content outcomes include helping children learn:

- to relate parts and the whole—part + part = whole and whole – one part = other part,
- to recognize when addition, and subtraction can be used to solve everyday situations,
- to represent those situations with models and symbols for addition and subtraction,
- to understand equals and the equals sign,
- to make sense of and spontaneously use a variety of reasoning strategies to solve problems,
- to make strategic choices about which reasoning strategy might be efficient, and
- to develop flexibility and fluency with addition and subtraction.

THINKING WITH NUMBERS

Lesson Descriptions

Explore With Word Problems

Practical everyday situations provide the context for understanding addition and subtraction. Each of the 11 different problem structures, as identified by Cognitively Guided Instruction, is explored in this section of lessons. Children solve word problems, create word problems, and after animations to illustrate the structure, figure out the unknown amount that is hidden under a cloud.

Expected content outcomes include helping children learn:

- solve problems for each of the 11 different problems structures for addition and subtraction in the context of word problems about everyday situations and animations that illustrate the structure,
- to represent those situations with models and symbols for addition and subtraction,
- to create and solve word problems to match given conditions, and
- to recognize when addition, and subtraction can be used to solve everyday situations.

THINKING WITH NUMBERS

Lesson Descriptions

The Ten Frame

These lessons encourage children to use the ten frame to recognize relationships among numbers to ten and beyond. The number of empty boxes provides as much information as the number of filled boxes. The number of filled boxes and the number of empty boxes is always 10. Number relationships also become evident. It just takes 2 more than 5 to make 7. Or if you take 2 away from 7, there are 5.

Expected content outcomes include helping children learn:

- to recognize numbers without counting,
- to compare and relate numbers to ten,
- put parts together to make a whole, and
- to decompose the whole into parts.

Joining

Joining actions have almost become synonymous addition and certainly are children's initial encounters with addition. Joining actions match directly with putting two parts together to make a whole.

Expected content outcomes include helping children learn:

- to recognize joining can be represented by addition,
- to use numbers and the plus sign to represent addition, and
- to recognize when addition can be used to represent joining situations in everyday life.

THINKING WITH NUMBERS

Lesson Descriptions

Describing Parts & Whole

Describing two parts and considering the whole, without action, is another everyday situation that can be represented by addition. For example, asking how pieces of fruit are in a bowl when there are 3 bananas and 4 pears is a part + part = whole situation with no action. Describing the parts and asking how many are in the whole involves addition.

Expected content outcomes include helping children learn:

- to recognize describing parts and the whole can be represented by addition,
- to use numbers and the plus sign to represent addition, and
- to recognize when addition can be used to represent describing parts and the whole situations in everyday life.

Equals With Addition

Children often have misconceptions about the concept of equals. To help address these issues, an introduction to the equals sign has been separated from the introduction of the plus sign. For example, some children believe that the equals sign means to do something. Now it is time to add or time to subtract. Some children believe that 2 + 3 = 5 is correct, but 5 = 2 + 3 is not. Children commonly see a number sentence like 2 + __ = 5 and complete the blank by filling in 7. They see the + sign and the two numbers then just add. The equals sign means that the numbers on both sides are the same. 2 + __ is the same number as 5. Introduction to the equals sign is closely related to children's recognition that numbers can be represented in different ways. For example, 5 + 2, 1 + 6, 8 – 1, and 2 + 2 + 2 + 1 are just different ways to represent the number we usually write as 7.

Expected content outcomes include helping children learn:

- to understand that the numbers on each side of the equals sign are the same,
- to know that numbers can be represented in many ways,
- to know what part to add to another to make it the same number as the number on the other side of the equals sign, and
- to use numbers with the plus and equals signs to represent addition.

Separating

Separating actions, or take away, have almost become synonymous with subtraction and are children's initial encounters with subtraction. Separating actions are more difficult for children than joining actions because the whole is not as evident after part of it has been separated. For example, if you start with 5 and separate 3, children see 2 and 3, not 5. The 3 that has been separated is part of the 5. It takes time for children to learn to represent this situation as 5 – 3. Subtraction is also difficult for two other reasons. First, the language does not help children develop relationships with the prior knowledge they have for addition. Second, getting the answer is more difficult because counting back is much more difficult than counting up. Children with lots of experience with numbers and their partitions find subtraction to be easier to learn.

Expected content outcomes include helping children learn:

- to recognize separating can be represented by subtraction,
- to use numbers and the minus sign to represent subtraction, and
- to recognize when subtraction can be used to represent separating situations in everyday life.

Describing Missing Parts & Whole

Describing a part and the whole and considering the other part, without action, is another everyday situation that can be represented by subtraction. These situations involve missing parts. For example, asking how bananas are in a bowl when there are 7 pieces of fruit and 4 pears is a whole - part = part situation with no action. Describing a part and the whole and asking how many are in the other part involves subtraction.

Expected content outcomes include helping children learn:
- to recognize describing parts and the whole can be represented by subtraction,
- to use numbers and the minus sign to represent subtraction, and
- to recognize when subtraction can be used to represent describing parts and the whole situations in everyday life.

Equals With Subtraction

Children often have misconceptions about the concept of equals. To help address these issues, an introduction to the equals sign has been separated from the introduction of the minus sign. For example, some children believe that the equals sign means to do something. Now it is time to add or time to subtract. Some children believe that 8 - 3 = 5 is correct, but 5 = 8 - 3 is not. Children commonly see a number sentence like __ - 3 = 5 and complete the blank by filling in 2. They see the - sign and the two numbers then just subtract. The equals sign means that the numbers on both sides are the same. __ - 3 is the same number as 5. Introduction to the equals sign is closely related to children's recognition that numbers can be represented in different ways. For example, 9 - 4, 2 + 3, 6 – 1, and 2 + 2 + 1 are just different ways to represent the number we usually write as 5.

Expected content outcomes include helping children learn:

- to understand that the numbers on each side of the equals sign are the same,
- to know that numbers can be represented in many ways,
- to know what part to subtract from another to make it the same number as the number on the other side of the equals sign, and
- to use numbers with the minus and equals signs to represent subtraction.

Equalizing

These lessons will help children develop flexibility in their thinking. For example, if there are 6 on one side of the equals sign and 8 on the other, children are asked to figure out how they can make the numbers the same. They could add 2 to the 6 and make both sides 8; they could subtract 2 from the 8 and make both sides 6; they could take one from the 8 and give it to the 6 and make both sides 7; They could add 4 to the 6 and 2 to the 8 and make both sides 10; they could subtract 1 from the 6 and 3 from the 8 and make both sides 5.

Expected content outcomes include helping children learn:

- to understand that the numbers on each side of the equals sign are the same,
- to know that numbers can be represented in many ways, and
- to develop flexibility in thinking.

Joining With The Change Unknown

Some everyday situations involve joining, but you do not know both parts. For example, suppose you had $8 in a piggy bank and your grandmother told you she added to it. If you have $10 now, you can figure out that she added $2. This problem can be represented by 8 + __ = 10. These problems are difficult for children because they know to start with 8, but do not know what to add. The solution process, although there is joining action, is a subtraction process because you know the whole and one part and are trying to find the other part. This problem can also be represented by 10 – 8 = __. Children will recognize that counting up, using ten, or using known facts are often more efficient than counting to find the answer. These problems are also a perfect opportunity for children to recognize relationships among parts and the whole and between addition and subtraction.

part 1 + part 2 = whole
part 2 + part 1 = whole
whole – part 1 = part 2
whole – part 2 = part 1

Expected content outcomes include helping children learn:

- to recognize joining can be represented by addition, but also by subtraction,
- to use numbers, the plus or minus sign, and equals signs to represent a joining situation with both addition and subtraction number sentences, and
- to recognize these missing part situations in everyday life.

THINKING WITH NUMBERS

Lesson Descriptions

Joining With The Start Unknown

Some everyday situations involve joining, but you do not know both parts. For example, suppose you had some money in a piggy bank and your grandmother told you she added $8. If you have $10 now, you can figure out that you started with $2. This problem can be represented by __ + 8 = 10. These problems are difficult for children because it is a joining problem, which makes them think addition, but they do not know what to start with. The solution process, although there is joining, is a subtraction process because you know the whole and one part and are trying to find the other part. This problem can also be represented by 10 – 8 = __. Children will recognize that counting up, using ten, or using known facts are often more efficient than counting to find the answer. These problems are also a perfect opportunity for children to recognize relationships among parts and the whole and between addition and subtraction.

part 1 + part 2 = whole
part 2 + part 1 = whole
whole – part 1 = part 2
whole – part 2 = part 1

Expected content outcomes include helping children learn:

- to recognize joining can be represented by addition, but also by subtraction,
- to use numbers, the plus or minus sign, and equals signs to represent a joining situation with both addition and subtraction number sentences, and
- to recognize these missing part situations in everyday life.

THINKING WITH NUMBERS

Lesson Descriptions

Separating With The Change Unknown

Some everyday situations involve separation, but you do not know how many were taken away. For example, suppose you had 6 cookies and your dog ate some of them. If you have 4 cookies now, you can figure out that the dog ate 2 cookies. This problem can be represented by 6 - __ = 4. These problems are difficult for children because they know to start with 6, but do not know what to subtract. This problem can also be represented by 6 – 4 = __. The relationship between the whole and a missing part can be represented by a change unknown or a result unknown. It also can be represented by 4 + __ = 6. Children will recognize that counting up, using ten, or using known facts are often more efficient than counting to find the answer. These problems are a perfect opportunity for children to recognize relationships among parts and the whole and between addition and subtraction.

part 1 + part 2 = whole
part 2 + part 1 = whole
whole – part 1 = part 2
whole – part 2 = part 1

Expected content outcomes include helping children learn:

- to recognize separating can be represented by subtraction, but also by addition,
- to use numbers, the plus or minus sign, and equals signs to represent a separating situation with both addition and subtraction number sentences,
- to recognize these missing part situations in everyday life.

Separating With The Start Unknown

Some everyday situations involve separation, but you do not know how many you started with. For example, suppose you had some money in your wallet. If you spend $6 and later find that you still have $3, you can figure out that you started with $9. This problem can be represented by __ - 6 = 3. These problems are difficult for children because they know it is subtraction, but do not know where to start. The solution process, although there is separation, is an addition process because you know both parts and are trying to find the whole. This problem can also be represented by 6 + 3 = __. Children will recognize that counting up, using ten, or using known facts are often more efficient than counting to find the answer. These problems are a perfect opportunity for children to recognize relationships among parts and the whole and between addition and subtraction.

part 1 + part 2 = whole
part 2 + part 1 = whole
whole – part 1 = part 2
whole – part 2 = part 1

Expected content outcomes include helping children learn:

- to recognize separating can be represented by subtraction, but also by addition,
- to use numbers, the plus or minus sign, and equals signs to represent a separating situation with both addition and subtraction number sentences,
- to recognize these missing whole situations in everyday life.

THINKING WITH NUMBERS

Lesson Descriptions

The Number Line

The number line is more than a sequence of numbers. It is constructed with a starting point, zero, and a unit of length that represents one. Each number is one unit from the previous number; that is, 3 is 3 units from zero and 4 is 4 units from zero. By including distance in the representations of numbers, the knowledge about which numbers are close can be used in many ways. For example, using nice numbers that are close makes it easier to solve problems; estimating is making use of numbers that are close. These lessons also encourage children to make sense of the number line as a way to represent addition and subtraction with lengths. The number sequence and benchmark numbers, like multiples of ten, will help them expand their knowledge of number relationships to tens and beyond. Two more than 8 is ten, but two more than 58 is 60; two more than 98 is 100; two more than 138 is 140.

Expected content outcomes include helping children learn:

- to recognize numbers on the number line,
- to understand the unit length between each number,
- to understand how numbers are related to benchmark numbers,
- to represent addition by putting parts (lengths) together to make a whole, and
- to represent subtraction by decomposing the whole into parts (lengths).

Comparing With The Difference Unknown

Addition and subtraction can also be used in comparison situations. When two sets are compared, part of one can be matched to the other. The extras are the difference between the two sets. Similarly, length can be used to compare two numbers. Part (length) of one number can be matched to the other number. Any "extra" length is the difference between the two numbers. By using the number line to represent numbers, the difference between the numbers is simply the number of units (length) between them. For example, suppose $7 is compared to $9. It takes $2 more to be added to the $7 to have as much as $9, so the difference is $2. These problems are also a perfect opportunity for children to recognize relationships among small set, the large set and the difference and how both addition and subtraction can be used to represent comparison situations.

small set + difference = large set
difference + small set = large set
large set – small set = difference
large set – difference = small set

Expected content outcomes include helping children learn:

- to recognize the relationship among small set, large set, and the difference,
- to recognize comparing can be represented by both addition and subtraction,
- to use numbers, the plus or minus sign, and equals signs to represent a comparison situation with both addition and subtraction number sentences, and
- to recognize these comparison situations in everyday life.

Comparing With The Large Set Unknown

Addition and subtraction can also be used in comparison situations. When two sets are compared, part of one can be matched to the other. The extras are the difference between the two sets. Similarly, length can be used to compare two numbers. Part (length) of one number can be matched to the other number. Any "extra" length is the difference between the two numbers. By using the number line to represent numbers, when the difference is added to the small set, you match the large set. For example, suppose you have $7 and the large set is $2 more. Then you can add $2 the $7 to match the large set, $9. These problems are also a perfect opportunity for children to recognize relationships among small set, the large set and the difference and how both addition and subtraction can be used to represent comparison situations.

small set + difference = large set
difference + small set = large set
large set – small set = difference
large set – difference = small set

Expected content outcomes include helping children learn:

- to recognize the relationship among small set, large set, and the difference,
- to recognize comparing can be represented by both addition and subtraction,
- to use numbers, the plus or minus sign, and equals signs to represent a comparison situation with both addition and subtraction number sentences, and
- to recognize these comparison situations in everyday life.

THINKING WITH NUMBERS

Lesson Descriptions

Comparing With The Small Set Unknown

Addition and subtraction can also be used in comparison situations. When two sets are compared, part of one can be matched to the other. The extras are the difference between the two sets. Similarly, length can be used to compare two numbers. Part (length) of one number can be matched to the other number. Any "extra" length is the difference between the two numbers. By using the number line to represent numbers, when the difference is taken from the large set, you match the small set. For example, suppose you have $9 and the small set is $2 less. Then you can take $2 away from the $9 to match the small set, $7. These problems are also a perfect opportunity for children to recognize relationships among small set, the large set and the difference and how both addition and subtraction can be used to represent comparison situations.

small set + difference = large set
difference + small set = large set
large set – small set = difference
large set – difference = small set

Expected content outcomes include helping children learn:

- to recognize the relationship among small set, large set, and the difference,
- to recognize comparing can be represented by both addition and subtraction,
- to use numbers, the plus or minus sign, and equals signs to represent a comparison situation with both addition and subtraction number sentences, and
- to recognize these comparison situations in everyday life.

Too Much, …Too Little Information

This description will be coming soon.

THINKING WITH NUMBERS

Lesson Descriptions

Reasoning Strategies

Addition and subtraction involves much more than counting to get the answer. The reasoning strategies that go beyond counting enable children to efficiently, flexibly, and fluently use addition and subtraction in everyday life. Strategies that help children make sense of basic facts include: counting on to add, counting back to subtract, counting up to subtract, using ten to add and subtract, and using known facts to add and subtract. Each of these strategies can be used with larger numbers as well. Strategies like using ten and using known facts provide a solid base for mental math. Additionally, using nice numbers, like multiples of tens, adds to children's repertoire of mental math strategies. Estimating using front-end numbers, nice numbers, bounds, and rounding provide multiple ways for children to use numbers. It takes time for children to make sense and then to integrate these new thinking strategies into their solution processes for math problems. These lessons use animation to concretely illustrate each new thinking process and provide enough lessons to ensure success. Reasoning strategies are the gateway to number sense.

Expected content outcomes include helping children learn:

- multiple reasoning strategies for solving addition and subtraction problems,
- to develop flexibility and fluency with the use of these reasoning strategies,
- to confidently master basic facts, and
- to confidently use addition and subtraction in everyday life.

Counting On To Add

Instead of counting from one each time, it is often much easier to start with the larger part and count on to find the total. For example, to add 2 + 8, you can start with 8 and count on two more, 9, 10. This is especially efficient when one of the numbers you are adding is small, even when the other number is much larger. For example, counting on enables you to solve 67 + 2 almost as easily as 7 + 2. It enables children to quickly solve almost all of the basic facts to twelve.

Expected content outcomes include helping children learn:

- to make sense of counting on to add,
- to recognize that it is more efficient to start with the larger part,
- to recognize that counting on is strategically efficient when one of the numbers you are adding is small, and
- to become proficient in using counting on.

Counting Back To Subtract

Instead of counting the part you subtract, then counting what is left, it is often much easier to start with the whole and count back. For example, to subtract 7 – 2, you can start with 7 and count back two, 6, 5. This is especially efficient when the number you are subtracting is small, even when the whole is much larger. For example, counting back enables you to solve 86 – 2 almost as easily as 6-2.

Expected content outcomes include helping children learn:

- to make sense of counting back to subtract,
- to recognize that counting back is strategically efficient when the number you are subtracting is small, and
- to become proficient in using counting back.

Counting Up To Subtract

Instead of counting the part you subtract, then counting what is left, it is often much easier to start with the part you are subtracting and count up to the whole while keeping track of how many more you counted. For example, to subtract 10 – 8, you can start with 8 and count up to ten, 9, 10. That's just 2 more. This is especially efficient when the number you are subtracting is about the same size as the whole, even when the whole is much larger. For example, counting back enables you to solve 78 – 76 almost as easily as 8-6.

Expected content outcomes include helping children learn:

- to make sense of counting up to subtract,
- to recognize that counting up is strategically efficient when the number you are subtracting is about the same size as the whole, and
- to become proficient in using counting up to subtract.

Using Tens To Add

Addition problems, with the whole greater than ten, are difficult for children. It is often much easier to add to ten, then add the rest. For example, you can add 6 to 9 by starting with 9, then 1 more is 10, and 5 more is 15. This is especially efficient when one of the numbers you are adding is close to ten. It is just as efficient with larger numbers when one of the numbers is close to a multiple of ten. For example, using ten enables you to solve 38 + 5 is almost as easily as 8 + 5.

Expected content outcomes include helping children learn:

- to make sense of using ten to add,
- to recognize that using ten is strategically efficient when one of the numbers you are adding is close to ten or a multiple of ten, and
- to become proficient in using ten to add.

Using Tens To Subtract

Subtraction problems, with the whole greater than ten, are difficult for children. It is often much easier to use ten as a stepping stone. There are three different ways to do this.

First, you can start with the whole, then subtract to ten, then subtract the rest of the part being subtracting. For example, to solve 13 – 8, you can start with 13 and subtract 3 to get 10. Then you can subtract the rest of the 8, five less is 5.

Second, you can start with the whole and subtract 3 to get ten, then subtract enough to get to 8. By keeping track of how many you subtracted, you have the answer. For example, to solve 13 – 8, you can start with 13 and subtract 3 to get 10. Then you can subtract enough to get to 8. Two less is 8. You subtracted 3, then 2, so you subtracted 5.

Third, you can start at 8 and add up to ten, then add enough to get 13. By keeping track of how many you added, you know the answer. For example, to solve 13 – 8, you can start at 8. Two more is ten, then 3 more is 13. That's 5 more.

This is especially efficient when the number you are subtracting is close to ten. It is just as efficient with larger numbers when the number you are subtracting is close to a multiple of ten. For example, using ten enables you to solve 64 - 58 is almost as easily as 14 - 8.

Expected content outcomes include helping children learn:

- to make sense of using ten to subtract,
- to recognize that using ten is strategically efficient when the number you are subtracting is close to ten or a multiple of ten, and
- to become proficient in using ten to subtract.

Using Known Facts To Add

Addition problems, with the whole greater than ten, are difficult for children. It is often much easier to use a known fact that is close to the problem, then adjust to solve the problem. For example, since 6 + 7 is just one more than 6 + 6 and 6 + 6 is known to be 12, 6 + 7 is just one more than 12, or 13. Any known fact can be used, but doubles and tens are often used. It is just as efficient with larger numbers. For example, to solve 59 + 8, you can add 60 + 8. 59 + 8 is 1 less than 60 + 8, or 67. This is especially efficient when the known fact is within 1 or 2 of the problem.

Expected content outcomes include helping children learn:

- to make sense of using known facts to add,
- to recognize that using known facts is strategically efficient when the known fact is close to the problem, and
- to become proficient in using known facts to add.

Using Known Facts To Subtract

Subtraction problems, with the whole greater than ten, are difficult for children. It is often much easier to use a known fact that is close to the problem, then adjust to solve the problem. For example, since 15 - 8 is just one less than 16 - 8 and 8 + 8 is known to be 16, 15 - 8 is just one less than 8, or 7. Any known fact can be used, but doubles and tens are often used. It is just as efficient with larger numbers. For example, to solve 42 - 20, is just 2 more than 40 – 20 which is a known fact. So 42 – 20 is 22. This is especially efficient when the known fact is within 1 or 2 of the problem.

Expected content outcomes include helping children learn:

- to make sense of using known facts to subtract,
- to recognize that using known facts is strategically efficient when the known fact is close to the problem, and
- to become proficient in using known facts to subtract.

Use Nice Numbers

Each of the reasoning strategies for basic facts can be extended to mental math for use with larger numbers. Nice numbers, whether they are multiples of ten or other known facts, can also be used as known facts because adding and subtracting with them is easy. For example, to solve the problem 39 + 28, you can add 40 + 28, then adjust by subtracting 1. To solve 72 – 48, you can subtract 72 – 50, then adjust by adding 2 more. Any known fact can be used, multiples of ten and twenty-five are commonly used with larger numbers. These strategies enable children to solve problems with larger numbers mentally.

Expected content outcomes include helping children learn:

- to make sense of using known nice numbers to add and subtract mentally,
- to recognize that using known nice numbers is strategically efficient when the known fact is close to the problem, and
- to become proficient in using nice numbers to add and subtract mentally.

THINKING WITH NUMBERS

Lesson Descriptions

Change The Problem

For some problems, changing to a problem with the same answer often makes it easier. For example, to solve 43 + 29, you can change the problem by taking 1 from 43 and giving it to 29 to get 42 + 30. The answer is the same, but it is easier to add 42 + 30. Similarly, to solve 62-28, you can change the problem by adding two to each number and keeping the difference the same to get 64 – 30. This strategy enables children to solve problems with larger numbers mentally.

Expected content outcomes include helping children learn:

- to make sense of using change the problem to add and subtract mentally,
- to recognize that using change the problem is strategically efficient when the change makes the problem easier, and
- to become proficient in using change the problem to add and subtract mentally.

Choose An Efficient Strategy

After children have a repertoire of reasoning strategies, choosing an efficient strategy is an important skill. They may have multiple ways to solve the problem, but they need to choose one way that will be easy for them. There is no correct way. It simply has to be easy for them. For example, to solve the problem 9 + 5, you could count on, you could use ten, you could use a known fact, 10 + 5 = 15, then adjust, or you could change the problem to 10 + 4. Any of these strategies that make the problem easier is a good strategic choice.

Expected content outcomes include helping children learn:

- to recognize they know many different ways to solve addition and subtraction problems mentally,
- to become proficient in using many different reasoning strategies, and
- to become proficient in strategically choosing an efficient strategy for them to add and subtract mentally.

Estimating

There are many everyday situations involving addition and subtraction where an exact answer is not needed. An estimate is often enough information to answer the question. For example, suppose you buy two items that cost 43¢ and 47¢ and you only have $1. Do you have enough money to pay for them? Since both 43 and 47 are less than 50, 43 + 47 is less than 50 + 50, in this case 100¢. So without taking the time to add 43¢ and 47¢, you know that $1 is enough to purchase both items. There are several different reasoning strategies that can be used to estimate. These lessons focus on using front-end numbers, using nice numbers, using bounds, and using rounding. Any of these might be the most efficient way to estimate depending on the context and the numbers involved.

Expected content outcomes include helping children learn:

- to recognize they know many different ways to estimate the solutions to addition and subtraction problems mentally,
- to become proficient in using many different reasoning strategies to estimate, and
- to become proficient in strategically choosing an efficient strategy for them to estimate mentally.

Base Ten Blocks

Base ten blocks are a common and useful tool to represent numbers greater than ten. These lessons encourage children to make sense of two- and three-digit numbers in terms of tens and ones or hundreds, tens, and ones. These representations are useful for many computational procedures involving comparing, estimating, and adding and subtracting.

Expected content outcomes include helping children learn:

- to recognize and represent numbers with base ten blocks,
- to understand the ten times relationship between ten and ones, and between one hundred and tens,
- to understand how numbers are related to nice numbers (multiples of ten),
- to represent addition by putting tens and ones together to make a whole, and
- to represent subtraction by decomposing the whole into tens and ones then separating.

THINKING WITH NUMBERS

Lesson Descriptions

Using Front-End Numbers

Often when you can use an estimate for a computation, one of the easy ways to think is to use front-end numbers. For example, to estimate 42 + 65, since 40 + 60 is 100, the answer will be about 100. You might even note that the answer is slightly greater than 100. Depending on the context of the question you are trying to solve, that may be accurate enough to answer your question. For situations where the context of the question does not require a more accurate solution, ignoring all of the digits except the front-end digit can be very efficient.

Expected content outcomes include helping children learn:

- to make sense of using front-end numbers to estimate,
- to recognize that using front-end numbers to estimate can be strategically efficient, given the context of the question, and
- to become proficient in using front-end numbers to estimate.

Using Nice Numbers

Often when you can use an estimate for a computation, one of the easy ways to think is to use nice numbers that are close. For example, to estimate 73 – 26, since 73 is about the same as 75 and 26 is about the same as 25, you can simply subtract 75 – 25 to get 50. The answer is about 50. You might even note that the answer is slightly less than 50 because 73 is less than 75 and 26 is greater than 25. Depending on the context of the question you are trying to solve, that may be accurate enough to answer your question. For situations where the context of the question does not require a more accurate solution, using nice numbers that are close can be very efficient.

Expected content outcomes include helping children learn:

- to make sense of using nice numbers to estimate,
- to recognize that using nice numbers to estimate can be strategically efficient, given the context of the question, and
- to become proficient in using nice numbers to estimate.

Using Bounds

Often when you can use an estimate for a computation, you only need to know if the answer is greater than a lower bound, less than an upper bound, or sometimes if it is between a lower bound and an upper bound. For example, suppose a canoe has a limit of 350 pounds, two people who want to use that canoe might want to compare their total weight to 350. Your total weight needs to be less than 350 for the canoe to be safe. Sometimes you need to make sure the answer is more than a lower bound. For example, suppose you want to make two batches of cookies. It takes 1 cup of butter to make one batch. You have 3 cups of butter. In this case, 1 cup + 1 cup is the lower bound. You need at least that much butter to make two batches of cookies. Sometimes you need to know if the total is between two amounts. For example, suppose you need to send a package that has two items in it, one that weighs 4 pounds and another that weighs 7 pounds. If packages that weigh between 8 and 15 pounds can be sent for a special price, will your package qualify? In each of these cases, an accurate solution is not needed as long as the estimates indicate that the solution is within the bounds. Depending on the context of the question you are trying to solve, using an upper bound, a lower bound, or both may be an efficient way to answer the question.

Expected content outcomes include helping children learn:

- to make sense of using bounds to estimate,
- to recognize that using bounds to estimate can be strategically efficient, given the context of the question, and
- to become proficient in using bounds to estimate.

THINKING WITH NUMBERS

Lesson Descriptions

Using Rounding

Often when you can use an estimate for a computation, one of the easy ways to think is to use numbers that have been rounded to the nearest ten or hundred. For example, suppose you need to add 289 + 627. When rounded to the nearest hundred, the estimate becomes 300 + 600, so the sum is about 900. Depending on the context of the question you are trying to solve, that may be accurate enough to answer your question. For situations where the context of the question does not require a more accurate solution, using rounded numbers can be very efficient.

Expected content outcomes include helping children learn:

- to make sense of using rounded numbers to estimate,
- to recognize that using rounded numbers to estimate can be strategically efficient, given the context of the question, and
- to become proficient in using rounded numbers to estimate.

Symbolic Procedures

Sometimes you may need a written record of an exact computation. For example, suppose you need to buy some materials for home repair. You may need to add some amounts to find the total of how much to buy. A written record of these computations is often helpful. Fortunately, by combining or separating hundreds, tens, and ones, children will be able to make sense of symbolic procedures to add and subtract. The trading, 10 ones for 1 ten or 1 ten for 10 ones, we do with base ten blocks helps children make sense of the "carrying" and "borrowing" we use to construct the written record.

Expected content outcomes include helping children learn:

- to make sense of representing numbers with base ten blocks, then combining or separating the blocks to answer computational problems,
- to create a symbolic procedure by learning how to record the actions with the base ten blocks on paper, and
- to become reasonably proficient with symbolic procedures.

Addition

A written record for addition can be created by combining the ones, renaming 10 ones as 1 ten, then combining the tens. Renaming is often called "carrying." By combining and trading base ten blocks, children can make sense of these actions and learn to record the actions, that is, to create a written record called the symbolic procedure.

Expected content outcomes include helping children learn:

- to make sense of representing numbers with base ten blocks, then combining them to answer computational problems,
- to create a symbolic procedure by learning how to record the combining and trading actions with the base ten blocks on paper, and
- to become reasonably proficient adding numbers symbolically.

THINKING WITH NUMBERS

Lesson Descriptions

Subtraction

A written record for subtraction can be created by trading a ten for 10 ones if needed, then separating the ones, then separating the tens. The trading is often called "borrowing." By trading and separating base ten blocks, children can make sense of these actions and learn to record the actions, that is, to create a written record called the symbolic procedure.

Expected content outcomes include helping children learn:

- to make sense of representing numbers with base ten blocks, then separating them to answer computational problems,
- to create a symbolic procedure by learning how to record the trading and separating actions with the base ten blocks on paper, and
- to become reasonably proficient subtracting numbers symbolically.